Steady copropagation mode of laser pulse and plasma wave

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(Received 30 May 2002; revised manuscript received 23 January 2003; published 28 March 2003)

Stimulated Raman scattering is investigated using a nonperturbative method. The existence of steady copropagation modes of laser field and plasma wave is discussed. The plasma wave strength is analyzed through its relation with the total energy of the laser-plasma system. It is found that at a sufficiently high laser intensity there exist stable copropagating modes with non-zero-strength plasma waves. The strength of plasma wave in a stable steady copropagation mode is found to have an exact expression in terms of the frequency, the wave vector of the plasma wave and the laser intensity.

DOI: 10.1103/PhysRevE.67.036408 PACS number(s): 52.38.Bv, 52.38.Dx

I. INTRODUCTION

As a fundamental phenomenon in laser-plasma interaction, stimulated Raman scattering (SRS) has been intensively investigated over the last several decades. Now it is widely believed that SRS is an effective absorption mechanism of the laser in plasmas. The efficiency of this SRS absorption depends on the final amplitude of driven plasma wave, or the growth rate of plasma wave. Numerous theoretical analyses have been devoted to the spatial and the temporal growth of driven plasma wave [1–7]. Among these works, SRS is believed to be triggered by thermal density fluctuation within the plasma. Hence, the final amplitude of plasma wave depends on the amplitude of the initial thermal wave, the growth rate and the interaction time. Because of the uncertainty in the amplitude of the initial noise wave, it is difficult to estimate the effectiveness of SRS.

The dynamic effects of plasma wave on the laser field has been widely investigated in these works [1–7]. In particular, these investigations focused at the effect of a single wave with constant amplitude ahead of the laser pulse. This corresponds to the pulse experiencing a coherent collective oscillation throughout the whole plasma region. In thermal plasma, the disordered thermal motion of the electrons competes with coherent collective oscillation and hence, do not favor large-scale plasma wave. Hence, it is necessary to extend the investigation to several thermal waves with varying amplitude. In addition, the role of thermal source is to drive a plasma wave continuously. Whether the plasma wave can be maintained or not when thermal source is removed is also discussed.

Here, we consider the existence of a steady copropagation mode of a laser field and a plasma wave. Because of the response of plasma to the laser field, the propagation of the laser wave strongly couples with the plasma wave. In addition to the fundamental pump, it possible that some sideband (Stokes and anti-Stokes) components will copropagate with the fundamental pump. A steady copropagation mode of the laser field contains different components with fixed relative intensities accompanied by a steady copropagating plasma wave. This distribution of laser field over several components is a result of the response of plasma. This steady copropagation mode has definite energy and represents a free particle state of laser-plasma system. The monochromatic distribution can be viewed as a special copropagating mode characterized by zero plasma wave intensity.

The paper is arranged as follows. The theoretical model equations are developed in Sec. II. Numerical solutions and the discussion of the numerical results are described in Sec. III. Our results are summarized in Sec. IV.

II. THEORY AND FORMALISM

We start from a nonrelativistic Newton's equation in one-dimensional (1D) case

$$\partial_t v_z = \frac{1}{m_e}[\partial_z(a_0 a_+^* + a_0 a_-^*) - \partial_z \phi]. \tag{1}$$

Here, v_z and m_e are the longitudinal oscillating velocity and the rest mass of electron, respectively. ϕ is the scalar electric potential associated with the plasma wave. a_0, a_+, and a_- are the laser vector potential of the fundamental component, the anti-Stokes component and the Stokes one, respectively,

$$a_0 = \frac{1}{\sqrt{1+f_+^2+f_-^2}}\sqrt{I_0}\exp(iKz - iWt), \tag{2a}$$

$$a_+ = a_0 f_+ \exp(ikz - i\omega t), \tag{2b}$$

$$a_- = a_0 f_- \exp(-ikz + i\omega t), \tag{2c}$$

where K and W are the wave vector and the frequency of the fundamental component, k and ω are the relative shift in wave vector and frequency of the sideband component, respectively, f_+ and f_- are the relative amplitudes of sidebands. Here, we are interested in the steady distribution of laser field over the three components, f_+, as well as f_-, is therefore taken as an invariant independent of z and t.

Now we rewrite Eq. (1) as

$$\partial_t v_z = \frac{1}{m_e}[|a_0|^2 \partial_z \{f_+ \exp(ikz - i\omega t) + f_- \exp(-ikz + i\omega t)\} - \partial_z \phi]. \quad (3)$$

We introduce a parameter λ whose value remains to be determined and split the above equation into

$$\partial_t v_z - \lambda \frac{1}{m_e}|a_0|^2 \partial_z [f_+ \exp(ikz - i\omega t) + f_- \exp(-ikz + i\omega t)] = (1-\lambda)\frac{1}{m_e}|a_0|^2 \partial_z [f_+ \exp(ikz - i\omega t) + f_- \exp(-ikz + i\omega t)] - \frac{1}{m_e}\partial_z \phi. \quad (4)$$

Since f_+ and f_- are invariant in space and time, there is

$$\partial_z [f_+ \exp(ikz - i\omega t) + f_- \exp(-ikz + i\omega t)] = -\frac{k}{\omega}\partial_t [f_+ \exp(ikz - i\omega t) + f_- \exp(-ikz + i\omega t)]. \quad (5)$$

Hence, we can obtain the following relations:

$$\phi = (1-\lambda)|a_0|^2 [f_+ \exp(ikz - i\omega t) + f_- \exp(-ikz + i\omega t)], \quad (6)$$

$$v_z = -\lambda \frac{1}{m_e}|a_0|^2 [f_+ \exp(ikz - i\omega t) + f_- \exp(-ikz + i\omega t)]\frac{k}{\omega}. \quad (7)$$

Moreover, there is a continuity equation for the electronic fluid

$$\partial_t n + \partial_z n_0 v_z = 0, \quad (8)$$

where n is the density oscillation, n_0 is the static plasma density. n and ϕ are related with each other via the equation

$$[\partial_{tt} - \partial_{zz}]\phi = -n, \quad (9)$$

where the "$-$" in right-hand side of this equation is due to the negative charge of an electron.

Combining the above equations, we can determine the value of λ,

$$\lambda = \frac{(k^2 - \omega^2)\omega^2}{[(k^2 - \omega^2)\omega^2 + \omega_p^2 k^2]}, \quad (10)$$

where $\omega_p = \sqrt{n_0/m_e}$ is the plasma frequency. It depends on the values of k and ω. Physically, λ represents the response of plasma to the dynamical ponderomotive force. Note that λ does not explicitly depend on the relative amplitudes f_+ and f_-.

Among the three variables describing the laser-plasma system, only the laser vector potential a is independent and determines other two variables, n and v_z. This is a natural result because the three variables, a, n, and v_z, are described by three equations. The above analysis indicates that the plasma response to the periodic force, the electronic velocity and the density oscillation can be determined via two equations, Newton's equation and the continuity equation. Thus, once the periodic force is determined, we can exactly know the plasma response.

Because the appearance of the laser field will inevitably induce the plasma response, this response cause in turn the propagation of laser field exhibiting nonlinear feature. Especially, when the laser field is a multicolor one, the beat between the different frequency components provides a periodic force on the plasma. The propagation of this multicolor field is complicated because of the plasma response to the periodic force. We will derive a nonlinear propagation equation of the laser field via variation principle. For the laser-plasma system, its formal Hamiltonian and Lagrangian read

$$H = \Big[(|\partial_t a_0|^2 + |\partial_t a_+|^2 + |\partial_t a_-|^2) + (|\partial_z a_0|^2 + |\partial_z a_+|^2 + |\partial_z a_-|^2) + \frac{n_0}{m_e}(|a_0|^2 + |a_+|^2 + |a_-|^2)\Big] + n_0 m_e |v_z|^2 - \frac{1}{2}[n^* \phi + \text{H.c.}] + \frac{1}{2}\Big[\frac{1}{m_e}(n_+ a_0 a_+^* + n_- a_0 a_-^* + \text{H.c.})\Big] - |\partial_t \phi|^2 - |\partial_z \phi|^2, \quad (11)$$

$$L = 2(|\partial_t a_0|^2 + |\partial_t a_+|^2 + |\partial_t a_-|^2 - |\partial_t \phi|^2) - H, \quad (12)$$

where n_+ and n_- depend on f_+ and f_-, respectively, which can be known from the expression of ϕ. This Lagrangian is a combination of three Lagrangians $L = L_{laser} + L_\phi + L_{free}$, one for the laser field, one for the ϕ field, and one for the free electrons. As ϕ, v_z, and a are independent of each other, the variation principle will lead to three independent motion equations, where r is the coordinate of an electron,

$$0 = \frac{\delta L}{\delta a} = \frac{\delta L_{laser}}{\delta a} = (\partial_{zz} - \partial_{tt})a - \omega_p^2 a - j_\perp, \quad (13a)$$

$$0 = \frac{\delta L}{\delta \phi} = \frac{\delta L_\phi}{\delta \phi} = (\partial_{zz} - \partial_{tt})\phi - n, \quad (13b)$$

$$0 = \frac{\delta L}{\delta r} = \frac{\delta L_{free}}{\delta r} = -\partial_{tt} r + \nabla [j_\perp(r) a - \phi(r)]; \quad (13c)$$

the latter two equations correspond to Eq. (9) and Eq. (1), respectively, the first one describes the propagation of the laser field when the fluctuating current $j_\perp$ is present.

Substituting the expressions of n, ϕ, and $a_\pm$ into Hamiltonian, we obtain

$$H = (1 + f_+^2 + f_-^2)(W^2 + K^2 + \omega_p^2)|a_0|^2 + |a_0|^2(\omega^2 + k^2) \times (f_+^2 + f_-^2) + 2(W\omega + Kk)(f_+^2 - f_-^2)|a_0|^2 + |a_0|^4 \times [(1-\lambda)^2(k^2 - \omega^2) - (1-\lambda)(k^2 - \omega^2)](f_+^2 + f_-^2)$$

$$+\lambda^2\omega_p^2\frac{k^2}{\omega^2}|a_0|^4(f_+^2+f_-^2)-(\omega^2+k^2)(1-\lambda)^2|a_0|^4\times(f_+^2+f_-^2). \quad (14)$$

We use two parameters to describe the frequency shift and the wave vector shift because there may be a difference between them

$$\beta+\varepsilon=\frac{\omega}{W},\quad \beta-\varepsilon=\frac{k}{K}. \quad (15)$$

We can separately obtain the propagation equations of the three components via the variation principle $(\delta L/\delta a)=0$. Although v_z and a are two vectors perpendicular to each other, $v_z\cdot v_z^*$ and $a\cdot a^*$ are scalar quantities. Because the unit vectors $e_\perp$ and e_z satisfy $e_z\cdot e_z=e_\perp\cdot e_\perp=1$, we have $\delta(v_z e_z\cdot v_z^* e_z)/\delta(a^* e_\perp)=\delta(v_z e_\perp\cdot v_z^* e_\perp)/\delta(a^* e_\perp)=\delta(v_z\cdot v_z^*)/\delta a^* e_\perp$. The propagation equations of the three components are written as

$$0=\Big\{(1+f_+^2+f_-^2)(W^2-K^2-\omega_p^2)-2\lambda^2\omega_p^2\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}\frac{K^2}{W^2}|a_0|^2(f_+^2+f_-^2)+(W^2-K^2)(f_+^2+f_-^2)[2|a_0|^2(\lambda-1)(\beta^2+\varepsilon^2)+(\beta^2+\varepsilon^2)]+(W^2+K^2)(f_+^2+f_-^2)[4|a_0|^2(\lambda-1)\beta\varepsilon+2\beta\varepsilon]+2\beta(W^2-K^2)(f_+^2-f_-^2)+2\varepsilon(W^2+K^2)(f_+^2-f_-^2)\Big\}a_0, \quad (16a)$$

$$0=\Big\{(W^2-K^2-\omega_p^2)-2\lambda^2\omega_p^2\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}\frac{K^2}{W^2}|a_0|^2+(W^2-K^2)[2|a_0|^2(\lambda-1)(\beta^2+\varepsilon^2)+(\beta^2+\varepsilon^2)]+(W^2+K^2)[4|a_0|^2(\lambda-1)\beta\varepsilon+2\beta\varepsilon]+2\beta(W^2-K^2)+2\varepsilon(W^2+K^2)\Big\}a_0 f_+, \quad (16b)$$

$$0=\Big\{(W^2-K^2-\omega_p^2)-2\lambda^2\omega_p^2\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}\frac{K^2}{W^2}|a_0|^2+(W^2-K^2)[2|a_0|^2(\lambda-1)(\beta^2+\varepsilon^2)+(\beta^2+\varepsilon^2)]+(W^2+K^2)[4|a_0|^2(\lambda-1)\beta\varepsilon+2\beta\varepsilon]-2\beta(W^2-K^2)-2\varepsilon(W^2+K^2)\Big\}a_0 f_-. \quad (16c)$$

Here, the nonlinear equation of fundamental field becomes dependent on the strength of sideband. Although the equations of sidebands are still linear, their parameters change due to the variation in the fundamental field. In the standard perturbative theories in which each physical quantity expands as a series of small perturbation parameters, the fundamental field is treated as an invariant because of its independence of perturbation parameter. In our nonperturbative treatment, the back action of sideband on the fundamental field is taken into account and the fundamental field becomes dependent on the intensity of sideband.

Because of the difference between the last two equations in the above equation set, the solution of this equation set must satisfy the condition that f_+ and f_- cannot simultaneously be unequal to zero, i.e.,

$$f_+f_-=0. \quad (17)$$

Clearly, $(f_+,f_-)=(0,0)$ corresponds to the monochromic distribution. For $f_+=0$, we can obtain following relation from Eqs. (16a) and (16c):

$$0=-2\lambda^2\omega_p^2\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}\frac{K^2}{W^2}|a_0|^2+(W^2-K^2)[2|a_0|^2(\lambda-1)\times(\beta^2+\varepsilon^2)+(\beta^2+\varepsilon^2)]+(W^2+K^2)[4|a_0|^2(\lambda-1)\beta\varepsilon+2\beta\varepsilon]-2\beta(W^2-K^2)-2\varepsilon(W^2+K^2), \quad (18a)$$

$$0=W^2-K^2-\omega_p^2. \quad (18b)$$

Similarly, for $f_-=0$, there is

$$0=-2\lambda^2\omega_p^2\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}\frac{K^2}{W^2}|a_0|^2+(W^2-K^2)[2|a_0|^2(\lambda-1)\times(\beta^2+\varepsilon^2)+(\beta^2+\varepsilon^2)]+(W^2+K^2)[4|a_0|^2(\lambda-1)\beta\varepsilon+2\beta\varepsilon]+2\beta(W^2-K^2)+2\varepsilon(W^2+K^2), \quad (19a)$$

$$0=W^2-K^2-\omega_p^2. \quad (19b)$$

For SRS, $\beta+\varepsilon=\omega_p/W$. Combining the above two equations, we have a resonant condition for the sideband

$$0=\left\{-2\lambda^2\left[\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}-(\beta-\varepsilon)^2\right]+2(\lambda-1)(\beta^2+\varepsilon^2)+[4(\lambda-1)\beta\varepsilon]\left[\frac{2}{(\beta+\varepsilon)^2}-1\right]\right\}|a_0|^2+[(\beta^2+\varepsilon^2)-2\beta]+[2\beta\varepsilon-2\varepsilon]\left[\frac{2}{(\beta+\varepsilon)^2}-1\right]. \quad (20)$$

Once β and ε satisfy this resonant condition, the corresponding sideband (β,ε) can have nonzero strength. From this equation, we know that whether β and ε satisfy the resonant condition or not depends on $|a_0|^2$. With the sideband growing, the intensity of fundamental component $|a_0|^2$ will vary.

Thus, the relative position of the sideband to fundamental component, or β and ε, will change. Unless β and ε satisfy the following equations:

$$0=-2\lambda^2\left[\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}-(\beta-\varepsilon)^2\right]+2(\lambda-1)(\beta^2+\varepsilon^2)+[4(\lambda-1)\beta\varepsilon]\left[\frac{2}{(\beta+\varepsilon)^2}-1\right], \tag{21a}$$

$$0=[(\beta^2+\varepsilon^2)-2\beta]+[2\beta\varepsilon-2\varepsilon]\left[\frac{2}{(\beta+\varepsilon)^2}-1\right], \tag{21b}$$

the relative position of sideband will be invariant with $|a_0|^2$ changing. Whether a sideband can be resonant or not when the intensity of fundamental component changes is important to its growth. Suppose that a sideband (β,ε) satisfies Eq. (20) at $|a_0|^2=x_1$, it can grow at this $|a_0|^2=x_1$. However, with this (β,ε) sideband growing, $|a_0|^2$ will decrease to $x_2<x_1$, thus this (β,ε) sideband will be out of resonant and slowed down its growth if it does not satisfy Eq. (21).

The sidebands with (β,ε) satisfying Eq. (21) represent a type of particular excitation of laser field, which will be resonant with the fundamental component at any strength. For such sidebands, their strength can grow to a substantial level because the sideband always be resonant. Hence, (β,ε) satisfying Eq. (21) denotes the steady copropagation modes of laser field and plasma wave.

The relative positions of sidebands are determined by Eq. (21). Their positions are independent of the intensity of fundamental component. Now we determine the strength of the sideband. For simplicity in symbol, let $f_-=f$, $f_+=0$. We rewrite Hamiltonian as

$$\begin{aligned}H=\Bigg\{&\frac{2}{(\beta+\varepsilon)^2}I_0+\left[1+\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}-(\beta-\varepsilon)^2\right]\frac{f^2}{1+f^2}I_0\\&-2\left[\frac{1}{(\beta+\varepsilon)}+\frac{(\beta-\varepsilon)}{(\beta+\varepsilon)^2}-(\beta-\varepsilon)\right]\frac{f^2}{1+f^2}I_0\\&+(\lambda^2-\lambda)\left(1-\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}+(\beta-\varepsilon)^2\right)\frac{f^2}{(1+f^2)^2}I_0^2\\&+\lambda^2\left[\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}-(\beta-\varepsilon)^2\right]\frac{f^2}{(1+f^2)^2}I_0^2-(1-\lambda)^2\\&\times\left[1+\frac{(\beta-\varepsilon)^2}{(\beta+\varepsilon)^2}-(\beta-\varepsilon)^2\right]\frac{f^2}{(1+f^2)^2}I_0^2\Bigg\}\omega_p^2,\end{aligned} \tag{22}$$

where λ can be derived from Eq. (10),

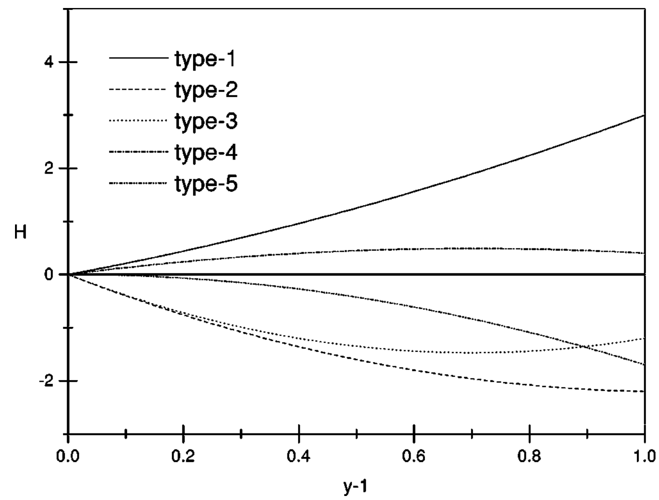

FIG. 1. Five possible types of H-z curve, where $z=1-y$.

$$\lambda(\beta,\varepsilon)=\frac{-[4\beta\varepsilon+(\beta^2-\varepsilon^2)^2](\beta+\varepsilon)^2}{-[4\beta\varepsilon+(\beta^2-\varepsilon^2)^2](\beta+\varepsilon)^2+(\beta+\varepsilon)^2[1-(\beta+\varepsilon)^2]}. \tag{23}$$

More explicitly, we express H as

$$H(y)=S_0(I_0,\beta,\varepsilon)+[S_1(I_0,\beta,\varepsilon)-S_2(I_0,\beta,\varepsilon)](1-y)+S_2(I_0,\beta,\varepsilon)(1-y)^2, \tag{24a}$$

$$0<y=\frac{1}{1+f^2}<1. \tag{24b}$$

From this expression, we know that the y value at the minimum of Hamiltonian $y_{\min}$ is determined by (I_0,β,ε). The minimum of Hamiltonian and $y_{\min}$ can be expressed in following five types for different S_2, S_1, S_0. For positive S_2,

$$H_{\min}=S_0-\frac{(S_1-S_2)^2}{4S_2}, \tag{25a}$$

$$0<y_{\min}=1-\frac{S_1-S_2}{2S_2}<1, \tag{25b}$$

or

$$H_{\min}=S_0, \tag{26a}$$

$$y_{\min}=1, \tag{26b}$$

or

$$H_{\min}=S_0+S_1, \tag{27a}$$

$$y_{\min}=0. \tag{27b}$$

Moreover, for negative S_2, $y_{\min}$ is 0 or 1. For clearness, we use Fig. 1 to illustrate different types of H-z curves. For these five types, only the type-3 has a finite nonzero local

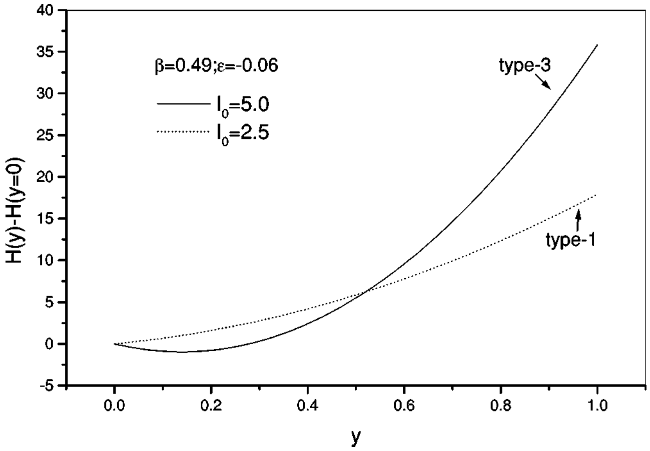

FIG. 2. The dependence of H on y when $\beta=0.49$, $\varepsilon=-0.06$.

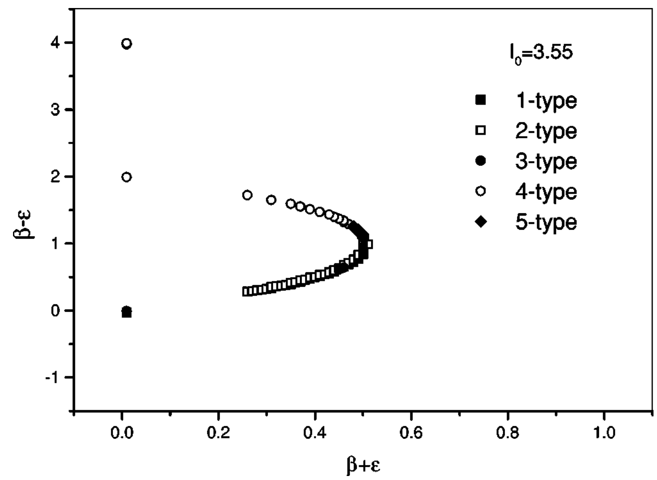

FIG. 3. The positions of $|a_0|^2$-independent resonant sidebands.

minimum $0<y_{\min}<1$. In other words, only type-3 mode can have nonzero plasma wave strength. For any sideband (β,ε), according to the values of S_2, S_1, S_0, we know which type this (β,ε) mode belongs to. When I_0 changes, the position of sideband (β,ε) does not change, but the type it belongs to may change. This can be illustrated by an example shown in Fig. 2. A mode $(\beta=0.49,\varepsilon=-0.06)$, which belongs to type-1 at low I_0 case, belongs to type-3 at high I_0 case.

For any (β,ε) mode, in principle, its strength can be arbitrary. However, only when its strength corresponds to $y_{\min}$, any small variation in strength will otherwise cause the energy of the system H to increase. We denote these modes as in LM state when their strength corresponds to a local minimum of H.

These steady copropagation modes are obtained from the variation principle. They represent free particle states of interacting laser-plasma system. If the system is isolated from the environment, it will keep its initial state with $f=0$, $(y=1)$. The environment contains numerous thermal sources, which can weakly perturb the laser-plasma system and exchange energy with the system. Here, the thermal source refers to the local electronic coherent oscillation and the disordered thermal motions of electrons, just not the density wave throughout the whole plasma. This coupling between the system and the environment cause the system energy to fluctuate, and the system state correspondingly oscillates between the different steady modes. On the other hand, we can take the system plus the environment as an isolated ensemble. For this ensemble, its energy, $E_{ens}=E_{sys}+E_{envir}$, is conserved. This ensemble has equal possibility to occupy every microscopic states of energy $E_{sys}+E_{envir}$. Thus, for the laser-plasma system, its possibility of occupying a steady state of energy E_1 depends on the number of the environment microscopic states of energy $E_{ens}-E_1$. For the environment, it consists of numerous thermal sources, and hence its energy can be approximately expressed as the summation of the energy of each thermal source, $E_{envir}=\Sigma_j e_j$. The microscopic state of the environment is described by the parameters of thermal sources, such as the energy of every thermal source. It should be noted that the larger E_{envir} is, the larger the number of the set (e_j) satisfying $E_{envir}=\Sigma_j e_j$. This suggests that larger E_{envir} corresponding to more microscopic states of the environment. Thus, when the degree of energy exchange between the system and the environment is given, smaller E_1, or larger E_{envir}, is favored.

From the above analyses, we know that LM states have a larger possibility to be present than that of their neighboring states whose f slightly differ from the LM state. Especially for a (β,ε) mode, when $f=0$ is not a LM state and a LM state with finite f exists, the system will tend to be present, for long time, plasma wave and (β,ε) sideband at a substantial level. Note that $f=\infty$ also corresponds to the plasma wave strength equal to zero.

III. NUMERICAL RESULTS AND DISCUSSIONS

In the following numerical experiments, we first present a 2D plot, Fig. 3 about the position of (β,ε) modes that satisfies $|a_0|^2$-independent resonant condition, Eq. (21). In Fig. 3, the mode with $\beta-\varepsilon$ larger than 1 corresponds to the sideband propagating in opposite direction to the fundamental component. Moreover, the modes in type-2 and type-5 correspond to monochromatic laser field in a new fundamental component, and hence the plasma wave strengths in such modes are zero. Because S_2 is square dependent on I_0, while S_1 is linear dependent on I_0, S_1-S_2 may change its sign and value when I_0 varies. In Fig. 4, the point near to the origin corresponds to very high fundamental frequency W, the numerical calculation of Hamiltonian indicates that it has very high energy exceeding that of the original monochromatic distribution. In contrast, the numerical results indicate that the points at the upper right corner have lower energies than that of the original monochromatic distribution. From Fig. 4, one can find that the type of a given mode may change when I_0 varies. With I_0 rising up to a threshold ~ 3.55, the number of type-3 modes increases. These results reveal that there are more possible sidebands that can appear in high I_0 case than those in low I_0 case. The existences of many type-3 modes indicates that even no explicit and definite external source drives, the system is still possible to keep a steady copropagation mode of laser field and plasma wave. The strength of

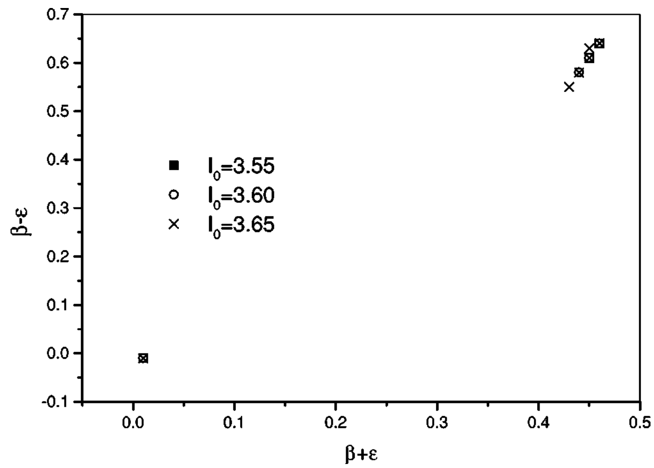

FIG. 4. The positions of type-3 (β,ε) modes under different I_0.

associated plasma wave in type-3 mode can be exactly calculated via Eq. (6), Eq. (9), and Eq. (25). The existence of type-3 mode implies that when the thermal source effect is not considered there exists a definite relation between the laser intensity and the plasma wave intensity. In contrast, the plasma wave driven by the thermal source does not have definite strength because of uncertainty of the initial strength of thermal seed. For realistic experimental condition, the strength of plasma wave is a mixture of the above two different classes, one is definitely dependent on the laser intensity and the other associated with the uncertain thermal seeds. Because the former requires sufficiently high laser intensity, only the latter is responsible for the strength of plasma wave at low I_0 case. This result proves the validity of attributing plasma wave to being triggered by the thermal sources at low I_0 case. Furthermore, our numerical results indicate that for modes in type-2, their $H_{\min}$ are higher than at least that of one mode in type-3. These results reveal that type-3 mode is more favorable to decrease the system energy than that of type-2 mode.

The existence of stable steady copropagation mode is important to the interaction of strong relativistic laser beam with plasma. For the beam with sufficiently high laser intensity, the filamentation effect will cause the laser beam to break into many filaments in transverse section. Every filament region has high laser intensity and its transverse structure can be ignored because its transverse section is very small. We can apply the above method to analyze the longitudinal excitation of laser field in a filament region. It is possible for a single filament region to occur spontaneously copropagation mode with nonzero plasma wave strength.

IV. SUMMARY

Based on a nonperturbative method, we discuss the existence of steady copropagation modes of laser field and plasma wave. The fundamental pump is not treated as rigid, and the back action of sideband on the fundamental pump is considered. This back action arises from the plasma response to the laser field. An analysis of the plasma response indicates that there is an exact relation among the laser vector potential, the electronic velocity and the density oscillations. This exact relation causes the equation of fundamental pump to depend on the strength of sideband. At a sufficiently high laser intensity, the mutual dependence of the fundamental pump and the sideband can lead to stable steady multicolor distributions to have a lower energy than monochromic distribution. Such stable multicolor distribution can maintain a plasma wave in a substantial level for a long time even when the thermal source is removed.

ACKNOWLEDGMENTS

This work is under the auspices of the 973 Project and the National Natural Science Foundation of China.

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